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On the Capture of Electrons by Swiftly Moving Electrified Particles.

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(Communicated by R. H. Fowler, F.R.S.—Received January 27, 1927.)

When an α-particle passes through matter it may capture an electron and continue on its way as a singly ionised helium atom. It is of interest to calculate the chance of such a capture on a classical basis and compare the results with the experimental data of Rutherford, Henderson, and Jacobsen.* Fowler† has calculated this chance by applying equilibrium statistical theory although the conditions do not very closely resemble equilibrium conditions. In the following paper the process of capture is considered in detail, and two cases are treated in which the three-body collision which is involved can be broken up into two successive two-body collisions.

* Rutherford, 'Phil. Mag.,' vol. 47, p. 276; Henderson, 'Roy. Soc. Proc.,' vol. 109, p. 157 (1925); Jacobsen, 'Nature,' June 19, 1926, p. 858.

† R. H. Fowler, 'Phil. Mag.,' vol. 47, p. 257; 'Proc. Camb. Phil. Soc.,' vol. 22, p. 253.

I.—*On the Capture of Electrons from Light Atoms by very Swift Particles.*

Suppose a particle of charge E moves with velocity V through matter containing N atoms per unit volume, each atom consisting of a nucleus of charge Z surrounded by electrons of charge e and mass m at distances r, and that V is large compared with the velocities of the electrons. The sequence of events in a capture must be like that indicated in diagram 1; corresponding numbers on the two paths represent simultaneous positions of the particle and the electron. First there is a close "collision" of the particle and the electron at

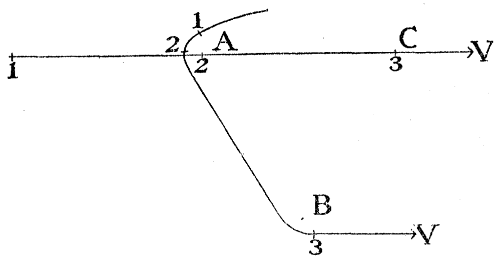

Diagram 1.

A, in which the electron has its small velocity altered nearly to V and so must go off at nearly 60° to the direction of motion of the particle; then the electron must pass near the nucleus at B and be deflected through nearly 60° to bring its direction of motion nearly parallel to that of the particle; at the same time the particle will have reached C, where ABC is nearly enough an equilateral triangle.

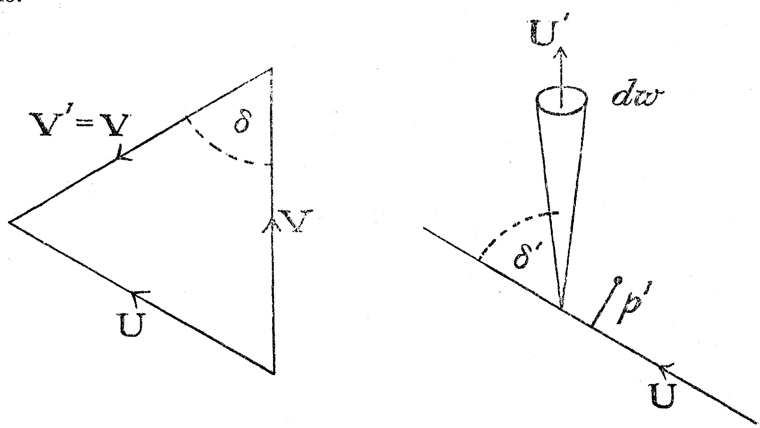

Diagram 2.

In travelling distance dl the particle has a chance

$$dW_1 = 2\pi p\, dp\, N\, dl$$

of passing an electron, at distance r from a nucleus, at a distance between p and $p + dp$, when the velocity of the electron relative to the particle will be deflected through angle δ given by

$$\cot \tfrac{1}{2}\delta = m\mathrm{V}^2 p/e\mathrm{E}.^* \tag{1.0}$$

If U is the velocity of the electron afterwards, $\mathrm{U} = 2\mathrm{V} \sin \frac{1}{2}\delta$ (diagram 2) so

$$p\,dp = \tfrac{1}{2} \operatorname{cosec}^3 \tfrac{1}{2}\delta \,.\, e^2\mathrm{E}^2/m^2\mathrm{V}^5 \,.\, d\mathrm{U}.$$

In order that $\mathrm{U} \simeq \mathrm{V}$,† $\delta \simeq \frac{1}{3}\pi$ and $\operatorname{cosec} \frac{1}{2}\delta \simeq 2$, so that

$$d\mathrm{W}_1 \simeq 2\pi \mathrm{N}\, dl \,.\, 4e^2\mathrm{E}^2/m^2\mathrm{V}^5 \,.\, d\mathrm{U}.$$

The chance that the electron, initially at distance r from the nucleus, will now pass it at a distance between p' and $p' + dp'$ and be deflected through angle δ' so as afterwards to move in a direction within a cone of angle $d\omega$ is (diagram 2)

$$d\mathrm{W}_2 = p'\,dp'\,d\phi/4\pi r^2 \left(= \frac{\text{target area}}{\text{area of whole sphere}}\right),$$

where

$$d\omega = \sin \delta'\, d\delta'\, d\phi, \quad \cot \tfrac{1}{2}\delta' = p' m\mathrm{U}^2/e\mathrm{Z},$$

so that

$$p'\,dp' = e^2\mathrm{Z}^2/m^2\mathrm{U}^4 \,.\, \tfrac{1}{2} \cot \tfrac{1}{2}\delta' \operatorname{cosec}^2 \tfrac{1}{2}\delta'\, d\delta'.$$

In order that the electron may afterwards travel in the direction of the particle, $\delta' \simeq \frac{1}{3}\pi$ and $\frac{1}{2}\cot \frac{1}{2}\delta' \operatorname{cosec}^2 \frac{1}{2}\delta' \simeq 4 \sin \delta'$ while $\mathrm{U} \simeq \mathrm{V}$, so that

$$d\mathrm{W}_2 \simeq \frac{4e^2\mathrm{Z}^2}{m^2\mathrm{V}^4} \frac{d\omega}{4\pi r^2}.$$

Further, if U′ is the final velocity of the electron, $\mathrm{U}' = \mathrm{U}$ and $d\mathrm{U}' = d\mathrm{U}$, so that the chance of both collisions is

$$\begin{aligned} d\mathrm{W} &= d\mathrm{W}_1\, d\mathrm{W}_2 \\ &\simeq 2\pi \mathrm{N}\, dl\ \frac{4e^2\mathrm{E}^2}{m^2\mathrm{V}^5}\, d\mathrm{U}'\ \frac{4e^2\mathrm{Z}^2}{m^2\mathrm{V}^4} \frac{d\omega}{4\pi r^2}. \end{aligned}$$

Just after this the electron is at distance r from the particle, moving in nearly the same direction and with nearly the same velocity as the particle ; in order that it should be captured, that is, should afterwards revolve about the particle, its velocity relative to the particle must be less than u, where

$$\tfrac{1}{2}mu^2 = e\mathrm{E}/r,$$

i.e.,

$$u = (2e\mathrm{E}/mr)^{\frac{1}{2}}.$$

* *Cf.* J. J. Thomson, 'Conduction of Electricity through Gases,' 2nd Ed., p. 376.

† In this paper the symbol "$\simeq$" is used to mean "is approximately equal to."

Thus the extremity of the vector U′ must lie within a sphere of radius u about the extremity of V,

i.e.,
$$U'^2 \, dU' \, d\omega = \tfrac{4}{3}\pi \, (2eE/mr)^{\frac{3}{2}}.$$

Hence the chance of a capture in path-distance dl of the particle

$$W \simeq 2\pi N \, dl \, \frac{4e^2E^2}{m^2V^5} \, \frac{4e^2Z^2}{m^2V^4} \, \frac{1}{4\pi r^2} \, \frac{1}{V^2} \, \tfrac{4}{3}\pi \left(\frac{2eE}{mr}\right)^{\frac{3}{2}}$$

$$\simeq \frac{64\sqrt{2}}{3} \, \pi a^2 N \, dl \left(\frac{Z}{e}\right)^2 \left(\frac{E}{e}\right)^{\frac{7}{2}} \left(\frac{a}{r}\right)^{\frac{7}{2}} \left(\frac{v}{V}\right)^{11}, \qquad (1.1)$$

where $av^2 = e^2/m$, say

$a = 5{\cdot}3 \,.\, 10^{-9}$ cm. the "radius of the normal orbit of a hydrogen atom."

$v = 2{\cdot}19 \,.\, 10^8$ cm./sec., the " velocity " of the electron in that orbit. (1.11)

This formula gives the chance of capture of an electron in the above way by a very fast particle. If the matter is gaseous, it should be summed over each electron together with each nucleus in a molecule; if solid, over each electron in an atom together with each neighbouring nucleus. It is really only applicable to α-particles moving in hydrogen or helium, and gives a very small chance of capture at the lowest speeds, say, $V = 4v = 8{\cdot}5 \,.\, 10^8$ cm./sec. at which it could be expected to hold. The experimental evidence, however, does not require that the chance should be any larger at these high velocities of the particle.*

Note.—The expression corresponding to (1.1) when the second collision is with another electron is

$$\tfrac{4}{3}\pi a^2 N \, dl \, (E/e)^{\frac{7}{2}} \, (a/r)^{\frac{7}{2}} \, (v/V)^{11}, \qquad (1.2)$$

and there is then also an equal chance of capturing the second electron. This can be found in the same way as (1.1), noting that the deflections of relative velocity in both collisions must now be approximately $\frac{1}{2}\pi$.

II.—*On the Capture of Electrons from Heavy Atoms by Swift Particles.*

A result was obtained in Part I by splitting up the three-body collision into two two-body collisions. This could be done because the speed of the particle was large compared with the initial speed of the electron. If the potential of the atomic field at the electron is large compared with the potential

* Rutherford, Jacobsen, *loc. cit.* Jacobsen has shown that this chance is much smaller in hydrogen than in air.

due to the particle at the same distance, this splitting up is possible even when the electronic speed is not small. Then also the particle must first pass close to the electron, compared with the latter's distance from the nucleus, to give the electron sufficient energy to escape from the nuclear field with speed near that of the particle; immediately after this close encounter, the relative velocity of the particle and the electron is large, and, since the atomic field is large compared with that of the particle, the effect of the particle on the motion of the electron may be neglected till both have left the neighbourhood of the nucleus.

An exact procedure would then be to use atomic fields and orbits such as those obtained by Hartree,* and to calculate on the above basis the chance of capturing each of the electrons in the atom for various velocities of the particle. For any particular electron, the chance would be small for a very swiftly moving particle, would increase as the particle's speed approached that of the electron, and would finally decrease when the particle's speed was so small that it could only with difficulty detach the electron from the atom at all. Thus, as the speed of the particle decreased, electrons would chiefly be captured from levels of lower and lower ionisation potential.

The calculation suggested would be little altered in result by replacing the discontinuous distribution of electrons in velocity, position, and ionisation potential, by a continuous distribution. An examination of Hartree's fields shows that the atomic fields are nearly inverse cube fields in the region relevant to the capture of electrons by α-particles of speed between $2 . 10^9$ and $4 . 10^8$ cm./sec. The correct distribution of electrons is two for each h^3 of volume of six-dimensional phase-space, for that part of phase-space with energy insufficient to carry the electron out of the atom.† This distribution does not correspond with reality at the outside of the atom, but the calculation in Part I shows that there the chance of capture is very small, and closer to the nucleus, where the field is no longer an inverse cube field, there are few electrons that can even be removed.

For comparison with experiment the chance of losing a captured electron is also required. It will appear that most electrons are not captured into closely bound orbits, and the time during which they remain with the particle is so short that they can hardly be supposed to fall into more closely bound orbits. Whether the electrons in the atoms must be regarded as having separate effects

* D. R. Hartree, 'Proc. Camb. Phil. Soc.,' vol. 21, p. 625. I am indebted to Mr. Hartree for the use of some unpublished fields.

† *Cf.* 'Proc. Camb. Phil. Soc.,' vol. 23, p. 542.

in detaching the captured electron from the particle, or whether the atoms can be supposed to act as wholes, will appear during the calculation.

Suppose a particle of charge E moves with velocity V through matter containing N atoms per unit volume. Each atom is represented by a central field in which an electron of charge e and mass m at distance r has potential energy $m\lambda/r^2$. The chance that there is an electron in volume $d\tau$ of the phase-space of that central field is $2d\tau/h^3$, where h is Planck's constant, so that the chance of an electron at distance between r and $r + dr$ and with velocity between v_2 and $v_2 + dv_2$ is

$$2 \,.\, 16\, \pi^2 \frac{m^3}{h^3} r^2\, dr\, v_2^{\,2}\, dv_2, \tag{2.1}$$

provided that

$$v_2^{\,2} < 2\lambda/r^2, \tag{2.11}$$

i.e., the electron is not in an orbit that leaves the atom. All directions for v_2 and r are independently equally likely.

When the particle has captured an electron, suppose it moves through matter containing M centres per unit volume, each centre being a field in which an electron has potential energy $m\nu/r^n$ at distance r; $n = 2$, $\nu = \lambda$, $\mathrm{M} = \mathrm{N}$, or $n = 1$, $\nu = e^2/m$, M some multiple of N, are to be taken according as the whole atom or separate electrons have a dominating effect, or possibly a combination of both.

Let $e\mathrm{E}/m = \mu$, so that an electron at distance r from a particle has potential energy $m\mu/r$.

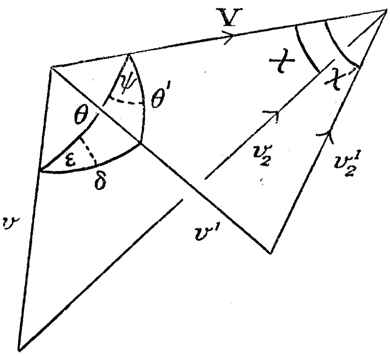

Diagram 3.

Suppose a particle collides with an electron of the group (2.1). All directions for v_2 are equally likely, and, v_2 fixed, the nucleus is equally likely to lie in any direction from the point of collision. Thus the chance that the angle $\widehat{v_2 \mathrm{V}}$ lies in a range $d\chi$ about χ is $\frac{1}{2}d\cos\chi$ and the orientation of the $(v_2\mathrm{V})$-plane about V makes no difference. χ, v_2, and V determine the relative velocity v of the particle and electron before the collision and the angle $\widehat{Vv}$ ($= \theta$). The relative orbit is then determined by the orientation of its plane about v, which may be measured from the (Vv)-plane and by the initial distance of the particle from the line of relative motion of the electron. If these lie in ranges $d\varepsilon$ and dp about ε and p respectively, the number of such collisions (diagram 3) in time dt will be

$$2.16\pi^2 \frac{m^3}{h^3} r^2\, dr\, v_2{}^2\, dv_2\, \tfrac{1}{2} d\cos\chi\ \mathrm{N} p\, dp\, d\varepsilon\, v\, dt.* \tag{2.2}$$

If u is the critical velocity of escape from the nucleus at distance r,

$$u^2 = 2\lambda/r^2, \tag{2.21}$$

and

$$r^2\, dr = (2\lambda)^{\frac{3}{2}}\, u^{-4}\, du,$$

further

$$dt = dl/\mathrm{V},$$

where dl is distance travelled by the particle, so (2.2) becomes

$$2.16\pi^2 \frac{m^3}{h^3} (2\lambda)^{\frac{3}{2}} \frac{du}{u^4} v_2{}^2\, dv_2\, \tfrac{1}{2} d\cos\chi\ \mathrm{N} p\, dp\, d\varepsilon \frac{v}{\mathrm{V}} dl. \tag{2.3}$$

The relative velocity after the collision, v', has the same magnitude as v but is deflected through angle δ given by

$$1 + p^2 v^4/\mu^2 = 2/(1 - \cos\delta) \quad (cf.\ (1.1)) \tag{2.31}$$

in the plane making angle ε with the (Vv)-plane. Then the angle $\widehat{Vv'}$ ($= \theta'$), the angle between the (Vv) and (Vv')-planes ($= \psi$), the velocity of the electron after the collision v_2', and the angle $\widehat{v_2'V}$ ($=\chi'$) are all determined. The independent variables p, χ and ε in (2.3) can be replaced by v_2', χ' and ψ.

$$p\, dp = \frac{\mu^2}{v^4} \frac{d\cos\delta}{(1 - \cos\delta)^2}. \qquad \text{from (2.31)}$$

Since θ, δ, θ' form a spherical triangle with ε, ψ as angles opposite θ' and δ respectively, for fixed χ, *i.e.*, fixed θ, v, and v',

$$d\cos\delta\, d\varepsilon = d\cos\theta'\, d\psi$$

$$= \frac{v_2'\, dv_2'}{v\mathrm{V}}\, d\psi \quad (\text{since } v_2'^2 = v^2 + \mathrm{V}^2 - 2v\mathrm{V}\cos\theta').$$

Then for fixed v_2' and ψ, since $v^2 = v_2{}^2 + \mathrm{V}^2 - 2v_2\mathrm{V}\cos\chi = v'^2 = v_2'^2 + \mathrm{V}^2 - 2v_2'\mathrm{V}\cos\chi'$,

$$d\cos\chi = \frac{v_2'}{v_2}\, d\cos\chi'.$$

* *Cf.* Jeans, 'Dynamical Theory of Gases,' p. 209.

Substituting these expressions, (2.3) becomes

$$2\,.\,16\pi^2\frac{m^3}{h^3}(2\lambda)^{\frac{3}{2}}\frac{du}{u^4}v_2^2\,dv_2\frac{v_2'}{2v_2}d\cos\chi'\;\frac{N\mu^2}{v^4}\frac{d\psi}{(1-\cos\delta)^2}\frac{v_2'\,dv_2'}{vV}\frac{v}{V}\,dl. \qquad (2.4)$$

For fixed u, v_2, χ', and v_2', ψ may have any value from 0 to 2π. For fixed u, v_2', and χ', and therefore fixed v, v_2 must satisfy the conditions

$$|v-V|<v_2<v+V$$

$$0<v_2<u. \qquad \text{from (2.11) and (2.21)}$$

If (2.4) is integrated over these values of ψ and v_2 the result will be the number of electrons that, in length dl of its path, a particle starts with velocity in range dv_2' about v_2' at an angle in range $d\chi'$ about χ' with the direction of motion of the particle at distances from nuclei corresponding to velocities of escape in range du about u. The nucleus is still equally likely to lie in any direction.

$$1-\cos\delta=1-\cos\theta\cos\theta'-\sin\theta\sin\theta'\cos\psi,$$

so

$$\int_0^{2\pi}\frac{d\psi}{(1-\cos\delta)^2}=\frac{2\pi(1-\cos\theta\cos\theta')}{|\cos\theta-\cos\theta'|^3}$$

$$=2\pi\left\{1-\frac{(v^2+V^2-v_2^2)}{2Vv}\frac{(v^2+V^2-v_2'^2)}{2Vv}\right\}\Big/\left|\frac{v_2'^2-v_2^2}{2Vv}\right|^3.$$

In order that capture may finally take place,

$$v_2'^2\simeq V^2+u^2 \qquad (2.41)$$

so

$$u<v_2'<v+V,$$

and the range of v_2^2 is $(v-V)^2<v_2^2<u^2$.

Replacing u^2 by $v_2'^2-V^2$,

$$\int_{|v-V|}^{u}\int_0^{2\pi}\frac{d\psi}{(1-\cos\delta)^2}v_2dv_2$$

$$\simeq\pi\int_{(v-V)^2}^{v_2'^2-V^2}\left[\left\{1-\frac{(v^2+V^2-x)}{2Vv}\frac{(v^2+V^2-v_2'^2)}{2Vv}\right\}\Big/\left(\frac{v_2'^2-x}{2Vv}\right)^3\right]dx.$$

$$\simeq\pi\frac{v}{V^3}[4V^2(v_2'^2-V^2)-(v_2'^2-v^2)^2].$$

The number of electrons so started is therefore

$$2\,.\,16\pi^3\frac{m^3}{h^3}(2\lambda)^{\frac{3}{2}}\frac{du}{u^4}\,N\mu^2\frac{d\cos\chi'}{2}v_2'^2\,dv_2'\,dl\,\frac{1}{v^3V^5}[4V^2(v_2'^2-V^2)-(v_2'^2-v^2)^2] \qquad (2.5)$$

where

$$v^2=V^2+v_2'^2-2Vv_2'\cos\chi',$$

provided that

$$u^2\simeq v_2'^2-V^2>(v-V)^2.$$

The orbit of the electron starting with velocity v_2' at distance r from the nucleus is determined by the direction in which the nucleus lies. The number of

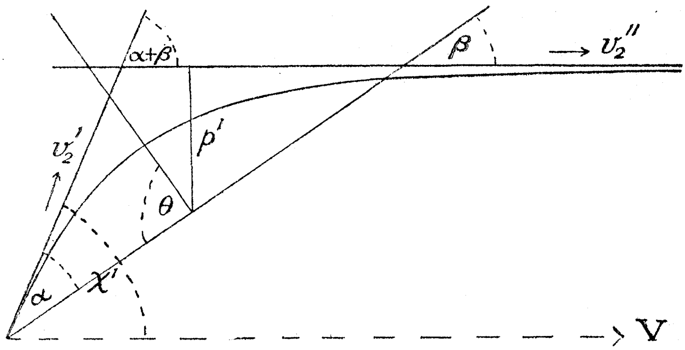

DIAGRAM 4.

electrons moving in orbits for which the angle $\widehat{rv_2'}$ lies in range $d\alpha$ about α, and the $(r\ v_2')$-plane makes angle in range $d\phi$ about ϕ with the $(v_2'\text{V})$-plane is obtained by multiplying (2.5) by

$$\frac{1}{4\pi} d\cos\alpha\, d\phi. \tag{2.51}$$

Let the angle between r and the asymptotic velocity of the electron, v_2'', say, be β (diagram 4). Then for fixed v_2' and α, β is fixed, and, varying χ' and ϕ, the solid angle traced out by the direction of v_2'' is

$$\sin(\alpha+\beta)\, d\phi \cos\phi\, d\chi'. \tag{2.52}$$

Now

$$v_2''^2 = v_2'^2 - u^2,$$

so, varying v_2',

$$v_2''\, dv_2 = v_2'\, dv_2'. \tag{2.53}$$

Combining (2.52) and (2.53), for varying v_2', ϕ, and χ' the extremity of the vector v_2'' traces out volume

$$\sin(\alpha+\beta)\, d\phi \cos\phi\, d\chi'\, v_2''\, v_2'\, dv_2'.$$

Let the final velocity of the electron relative to the particle have direction in solid angle $d\omega$ and magnitude in range dw about w. Changing independent variables fom v_2', ϕ, and χ' to w and the direction of w,

$$\sin(\alpha+\beta)\, d\phi \cos\phi\, d\chi'\, v_2''\, v_2'\, dv_2' = w^2\, dw\, d\omega. \tag{2.54}$$

In order that the electron may be captured, v_2'' must be nearly the same as V in magnitude and direction, *i.e.*,

$$v_2'' \simeq \text{V}$$
$$\alpha+\beta \simeq \chi'$$
$$\phi \simeq 0.$$

Using these approximations, from (2.5), (2.51) and (2.54), the number of electrons started where the velocity of escape is in range du about u in a direction making angle in range $d\alpha$ about α with the direction of the nucleus so as finally to have velocity relative to the particle in solid angle $d\omega$ and range dw about w is, for path length dl of the particle,

$$4\pi^2 \frac{m^3}{h^3} (2\lambda)^{\frac{3}{2}} \frac{du}{u^4} \mathrm{N}\mu^2 \, d\cos\alpha \, \frac{v_2' \, dl}{v^3 \mathrm{V}^6} [4\mathrm{V}^2 (v_2'^2 - \mathrm{V}^2) - (v_2'^2 - v^2)^2] \, w^2 \, dw \, d\omega, \quad (2.6)$$

provided that

$$u^2 > (v - \mathrm{V})^2.$$

If x is the distance between the electron and the particle after leaving the nuclear field, the condition of capture is

$$w^2 < 2\mu/x,$$

so the last part of (2.6) integrated for all captured electrons gives

$$\int w^2 \, dw \, d\omega = \tfrac{4}{3}\pi \left(\frac{2\mu}{x}\right)^{\frac{3}{2}}. \quad (2.61)$$

x and χ' must now be expressed in terms of α. If ρ, θ are polar co-ordinates in the orbit of the electron past the nucleus, θ measured from the apse, and p' is the distance of the asymptote from the nucleus,

$$\rho^2 \dot{\theta} = p'\mathrm{V} = r v_2' \sin\alpha$$

$$\rho^2 \dot{\theta}^2 + \dot{\rho}^2 = 2\lambda/\rho^2 + \mathrm{V}^2,$$

so

$$\theta = \int \left\{ \frac{1}{p'^2} - \frac{1}{\rho^2}\left(1 - \frac{2\lambda}{p'^2 \mathrm{V}^2}\right) \right\}^{-\frac{1}{2}} \frac{d\rho}{\rho^2}$$

$$= \left(1 - \frac{2\lambda}{p'^2\mathrm{V}^2}\right)^{-\frac{1}{2}} \cos^{-1} \frac{p'}{\rho} \left(1 - \frac{2\lambda}{p'^2\mathrm{V}^2}\right)^{\frac{1}{2}}$$

$$t = \int \left\{ \mathrm{V}^2 - \frac{1}{\rho^2} (p'^2\mathrm{V}^2 - 2\lambda) \right\}^{-\frac{1}{2}} d\rho$$

and

$$\mathrm{V}t = \left\{ \rho^2 - \left[p' \left(1 - \frac{2\lambda}{p'^2\mathrm{V}^2}\right)^{\frac{1}{2}} \right]^2 \right\}^{\frac{1}{2}}. \quad (2.62)$$

Thus

$$\pi + \beta = \left(1 - \frac{2\lambda}{p'^2\mathrm{V}^2}\right)^{-\frac{1}{2}} \sin^{-1} \frac{p'}{r} \left(1 - \frac{2\lambda}{p'^2\mathrm{V}^2}\right)^{\frac{1}{2}} \quad (2.63)$$

where on the right-hand side the inverse sine is greater than $\frac{1}{2}\pi$ for α less than $\frac{1}{2}\pi$ and *vice versa.*

From (2.62)

$$\mathrm{V}t = \rho + 0\left(\frac{1}{\rho}\right) \qquad \text{as } \rho \to \infty,$$

so that

$$\begin{aligned} x^2 &= (p' + r\sin\beta)^2 + (\{r^2 - p'^2(1 - 2\lambda/p'^2V^2)\}^{\frac{1}{2}} - r\cos\beta)^2 \\ &= p'^2 + r^2\sin^2\beta + 2p'r\sin\beta + r^2\cos^2\beta + r^2 - p'^2 + 2\lambda/V^2 \\ &\qquad\qquad - 2\cos\beta r(r^2 + 2\lambda/V^2 - p'^2)^{\frac{1}{2}} \\ &= 2r^2 + 2\lambda/V^2 + 2p'r\sin\beta - 2r^2v_2'/V\,.\,(1 - p'^2V^2/r^2v_2'^2)^{\frac{1}{2}}\cos\beta \\ &= 2r^2 + 2\lambda/V^2 + 2r^2v_2'/V\,.\sin\beta\sin\alpha - 2r^2v_2'/V\,.\cos\beta\cos\alpha \\ &= r^2/V^2\,.\,[2V^2 + 2\lambda/r^2 - 2Vv_2'\cos\chi] \\ &= r^2/V^2\,.\,v^2 \end{aligned}$$

and

$$x = \frac{r}{V}v = \frac{v}{V}\frac{\sqrt{2\lambda}}{u}. \tag{2.64}$$

Using (2.64) in (2.61) and the result in (2.6), the number of electrons captured by the particle in path distance dl is

$$dl\int_0^\infty\int_{(\alpha)}\frac{16}{3}\pi^3\frac{m^3}{h^3}N\mu^2(2\mu)^{\frac{3}{2}}(2\lambda)^{\frac{3}{4}}\frac{(u^2+V^2)^{\frac{1}{2}}}{u^{\frac{5}{2}}v^{\frac{9}{2}}V^{\frac{9}{2}}}[4V^2u^2 - (u^2+V^2-v^2)^2]\,d\cos\alpha\,du, \tag{2.7}$$

where

$$v^2 = 2V^2 + u^2 - 2V(V^2+u^2)^{\frac{1}{2}}\cos(\alpha+\beta)$$

$$\pi + \beta = \left(1 - \frac{2\lambda}{p'^2V^2}\right)^{-\frac{1}{2}}\sin^{-1}\left\{\left[\frac{p'^2u^2}{2\lambda}\left(1 - \frac{2\lambda}{p'^2V^2}\right)\right]^{\frac{1}{2}}\right\}$$

$$p'^2V^2 = 2\lambda/u^2.\,(u^2 + V^2)\sin^2\alpha,$$

and the integration with respect to α is over values of α between 0 and π for which v is real and u^2 less than $(v - V)^2$.

The number of electrons captured per unit path length is, therefore,

$$C\tfrac{16}{3}\pi^3m^3/h^3\,.\,N(2\mu)^{\frac{7}{2}}(2\lambda)^{\frac{3}{4}}V^{-\frac{11}{2}}, \tag{2.8}$$

where C is a numerical constant.

$$C = \int_0^\infty\int_{(\alpha)}\frac{(b^2+1)^{\frac{1}{2}}}{b^{\frac{5}{2}}e^{\frac{9}{2}}}[b^2 - \tfrac{1}{4}(b^2+1-e^2)^2]\,|\,d\cos\alpha\,|\,db, \tag{2.81}$$

where

$$e^2 = b^2 + 2 - 2\sqrt{b^2+1}\cos(\alpha+\beta)$$

$$\pi + \beta = (1 - f^2)^{-\frac{1}{2}}[\pi/2 \pm \cos^{-1}\{b\sqrt{1-f^2}/f\}]$$

and

$$f = b\,\mathrm{cosec}\,\alpha/\sqrt{1+b^2},$$

the positive sign is to be taken for α less than $\pi/2$, the negative sign for α greater than $\pi/2$. The integration with respect to α is for such values of α between 0 and π that the integrand is real (*i.e.*, $a\sqrt{1-f^2}/f<1$) and positive (*i.e.*, $(e-1)^2<a^2$).

The chance the particle loses an electron depends on how closely it has captured it. The proportion of the length a particle travels that it retains a captured electron is the same as the proportion of particles that have electrons at any moment. So long as this is small it can be obtained by integrating the product of (2.6) and the mean distance a particle retains an electron of speed w at distance x.

The deflection of an electron moving at speed V at distance p from a centre of force $mn\nu/r^{n+1}$ is

$$2\int_{r_0}^{\infty}\left\{\frac{1}{p^2}+\frac{2}{p^2V^2}\frac{\nu}{r^n}-\frac{1}{r^2}\right\}^{-\frac{1}{2}}\frac{dr}{r^2}-\pi,$$

where r_0 is the root of the quantity underneath the radical.*

For large V this deflection is asymptotically

$$\frac{2\nu}{V^2p^n}\frac{\Gamma\{\frac{1}{2}n+\frac{1}{2}\}\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2}n)} \qquad n>1,$$

so that the mean energy, referred to the particle, gained by the electron on passing at distance p from a centre is

$$\tfrac{1}{2}mV^2\left\{\frac{2\nu}{V^2p^n}\frac{\Gamma(\frac{1}{2}n+\frac{1}{2})\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2}n)}\right\}^2.$$

The particle loses the electron if this is greater than

$$\tfrac{1}{2}m\{2\mu/x-w^2\},$$

i.e., if

$$p<\left\{\frac{2\nu}{V}\frac{\Gamma(\frac{1}{2}n+\frac{1}{2})\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2}n)}\right\}^{\frac{1}{n}}\Big/\left(\frac{2\mu}{x}-w^2\right)^{\frac{1}{2n}}. \tag{2.85}$$

The mean distance the particle retains its electron is then

$$y=\left(\frac{2\mu}{x}-w^2\right)^{\frac{1}{n}}\Big/\pi M\left\{\frac{2\nu}{V}\frac{\Gamma(\frac{1}{2}n+\frac{1}{2})\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2}n)}\right\}^{\frac{2}{n}}.$$

When (2.6) is multiplied by this the integration with regard to w takes the form,

$$\int_0^{\left(\frac{2\mu}{x}\right)^{\frac{1}{2}}}y\,w^2\,dw=\frac{1}{\pi M}\left(\frac{2\mu}{x}\right)^{\frac{3}{2}+\frac{1}{n}}\frac{1}{2}\left(\frac{V\Gamma(\frac{1}{2}n)}{2\nu\Gamma(\frac{1}{2}n+\frac{1}{2})\Gamma(\frac{1}{2})}\right)^{\frac{2}{n}}\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(1+\frac{1}{n}\right)}{\Gamma\left(\frac{5}{2}+\frac{1}{n}\right)}. \tag{2.86}$$

If $V=2\,.\,10^9$ cm./sec., and $(2\mu/x)-w^2=(2\,.\,10^8$ cm./sec.$)^2$,

* *Cf.* J. J. Thomson, 'Conduction of Electricity through Gases,' 2nd Ed., p. 371.

then for an electron ($n = 1$, $\nu = e^2/m$) the maximum value of p in (2.85) is $1{\cdot}27\,.\,10^{-9}$ cm., while for a fairly heavy atom ($n = 2$, $\nu \simeq 10\,a\,e^2/m$), it is $1{\cdot}08\,.\,10^{-8}$ cm. Thus for $V = 2\,.\,10^9$, the atom as a whole will dominate the loss of electrons. For lower speeds of the particle, separate atomic electrons will have appreciable effects.

Using the same effective field as for capture ($n = 2$, $\nu = \lambda$, $M = N$) in (2.86),

$$\int_0^{\left(\frac{2\mu}{x}\right)^{\frac{3}{2}}} y w^2\, dw = \frac{1}{\pi N}\left(\frac{2\,\mu}{x}\right)^2 \frac{V}{16\,\lambda}.$$

Substituting in (2.6), replacing x by $v\sqrt{2\lambda}/Vu$, and reducing as before, the proportion of particles that at any moment have electrons is

$$C'\,2\,\pi^2 m^3/h^3\,.\,(2\mu)^4\,(2\lambda)^{-\frac{1}{2}}\,V^{-4}, \tag{2.9}$$

where C' is a numerical constant.

$$C' = \int_0^{\infty}\int_{(\alpha)} \frac{(b^2+1)^{\frac{1}{2}}}{b^2\,e^5}\,[b^2 - \tfrac{1}{4}(b^2+1-e^2)^2]\,|\,d\cos\alpha\,|\,db \tag{2.91}$$

over the same range in α as for C above.

Now let a be the "radius of the normal orbit of a hydrogen atom," v the "velocity" of the electron in that orbit, as in (1.11), so that

$$h = 2\pi e m^{\frac{1}{2}} a^{\frac{1}{2}}$$

$$v = e/m^{\frac{1}{2}} a^{\frac{1}{2}}.$$

Let

$$\lambda = \gamma a e^2/m,$$

so that γ is the ratio of the potential at distance a in the atomic field to that at distance a in a hydrogen atom. Then, since, as defined, $\mu = eE/m$, the results (2.8), (2.9) may be expressed as follows.

A particle of charge E moves with velocity V through matter containing N atoms per unit volume. The potential in the effective field of an atom at distance r is $\gamma a e/r^2$, and there are two electrons for each h^3 of volume in the phase-space of the motion of an electron in that field.

Then the chance that in distance dl the particle captures an electron is

$$C\tfrac{2}{3}\,(2E/e)^{\frac{7}{2}}\,(2\gamma)^{\frac{3}{4}}\,(V/v)^{-\frac{11}{2}}\,Na^2\,dl, \tag{3.1}$$

while the proportion of particles which at any moment have an electron is

$$C'\,\frac{1}{4\pi}\,(2E/e)^4\,(2\gamma)^{-\frac{1}{2}}\,(V/v)^{-4}, \tag{3.2}$$

where $C \simeq 3{\cdot}4$, $C' \simeq 4{\cdot}6$.

The values of C and C′ were found by a rough numerical integration. The inner integral, with respect to α, took the following values :

b	0·5	0·7	0·75	1	2
c	0·35	5·84	11·28	3·52	0·02
c'	0·45	7·08	15·25	4·55	0·02

in both cases there is a sharp maximum for $b = 0{\cdot}734$, where the condition limiting α changes its form.

Now

$$b = \frac{u}{V} = \sqrt{2\gamma}\,\frac{a}{r}\,\frac{v}{V},$$

so that for $2\gamma = 9$, say, and V/v varying from 10 to 3 the important part of the range of r/a is from 0·2 to 1·2. By comparison with Hartree's fields, the assumption as to the field is roughly justified, and for Na, K, Rb, Cs, 2γ has the values 6·2, 8·6, 10·6, 14·6.

The above formulæ agree with experiment as well as can be expected from the roughness of the assumptions on which they are based. The experimental data for these high-speed α-particles are roughly as follows. Rutherford found that the proportion of particles with an electron after passing through mica was 1/200 for $V = 1{\cdot}81 . 10^9$ cm./sec. and varied roughly as $V^{-4\cdot 6}$. Replacing mica by other solids made little difference. He measured also the rate of loss in air (and other gases). Assuming that the proportion is the same for air as for mica, the chance of capture in air (at N.T.P.) was 4·6 dl for $V = 1{\cdot}81 . 10^9$ cm./sec. and varied roughly as $V^{-5\cdot 6}$. Henderson found that for many solids ranging from mica to gold the proportion of particles was nearly the same and varied with velocity as V^{-n} where n decreased from 4·3 to 3·4 as V increased from $1{\cdot}0 . 10^9$ to $1{\cdot}35 . 10^9$ cm./sec. His actual value agrees with Rutherford's for about $V = 1{\cdot}5 . 10^9$ cm./sec.

Expression (3.2) makes the proportion of particles with an electron vary as V^{-4} and from substance to substance as $(2\gamma)^{-\frac{1}{2}}$, *i.e.*, very little. For $V = 1{\cdot}81 . 10^9$ cm./sec., taking $2\gamma = 8{\cdot}6$, $E = 2e$, the proportion calculated is 0·0064, Rutherford's observed value being 0·005.

Expression (3.1) makes the chance of capture vary as $V^{-5\cdot 5}$. Taking $V = 1{\cdot}807 . 10^9$ and $2\nu = 8{\cdot}6$, as above, and $N = 2 . 2{\cdot}705 . 10^{19}$, the chance of capture in, say, air at N.T.P. calculated is 20·2 dl, about four times as large as Rutherford's observed value. However, air is already so light that the assumptions are not very accurate.

Note.—The forms of results (1 . 1) (3 . 1) can be obtained, except for the numerical factor, by a dimensional argument, in the more general case of an atom attracting as an inverse nth power field.

A.—The chance W that an α-particle with velocity V attracting electrons with acceleration μ/r^2 will in distance dl capture an electron from atoms N per unit volume consisting of electrons at distance R from centres attracting with acceleration λ/r^n must be, by an argument similar to that given above, as a limiting form for large V,

$$\mathrm{W} = \text{const. } \mathrm{N}\mu^{\frac{7}{2}} \quad dl\, \mathrm{R}^{-\frac{7}{2}}\ \lambda^{-s}\mathrm{V}^{-t},$$

where

$$[\mathrm{N}] = \mathrm{L}^{-3}$$

$$[\lambda] = \mathrm{L}^{n+1}\,\mathrm{T}^{-2}$$

$$[\mu] = \mathrm{L}^{3}\,\mathrm{T}^{-2}$$

i.e.,

$$s = -\frac{2}{n-1},$$

$$t = 7 + \frac{4}{n-1}.$$

B.—If instead of electrons at distance R from the atomic centres there are electrons distributed at ρ per unit phase-space (mass of electron = unit mass),

$$\mathrm{W} = \text{const. } \mathrm{N}\mu^{\frac{7}{2}}\, dl\, \rho\, \lambda^{-s}\,\mathrm{V}^{-t}$$

where

$$[\rho] = \mathrm{T}^{3}\mathrm{L}^{-6}$$

so

$$s = -3/2\,(n-1),$$

$$t = 4 + 3/(n-1).$$

These results require for their validity that the conditions (magnitude of V, etc.) should allow the three-body collision to be split up into two two-body collisions as above, thus enabling the form of dependence of W on μ and R to be determined.

Finally I wish to express my appreciation of the encouragement and help of Prof. Bohr and Prof. Kramers while I was doing this work.

Summary.

The chance that a fast α-particle will capture an electron is calculated in two cases; for particles with velocity greater than $8 \cdot 10^8$ cm./sec. moving

through hydrogen or helium, and for particles with velocity between $4 . 10^8$ and $2 . 10^9$ cm./sec. moving through heavy matter. The method is to split up the process into a close collision of the particle with an electron and a close collision of this electron with an atomic nucleus. The results obtained are compared with the available experimental results and the agreement is satisfactory.

A further Contribution to the Study of the Phenomena of Intertraction.

By Sir Almroth E. Wright, M.D., F.R.S.

(Received January 13, 1927.)

[Plates 41, 42.]

Seeing that a phenomenon of lateral streaming which I recently described and put forward as convincing evidence of *horizontal intertraction* is construed otherwise by N. K. Adam,* I have, with a view to testing his interpretation of the phenomenon, made some further quite simple investigations. It will not be amiss, as a preliminary to detailing these, to place the real issue in debate—the issue as to whether there is such a *vis operans* as intertraction—before us in its proper perspective. I would propose to do this by recounting what led up to the investigation of intertraction.

The study of this *vis operans* began with observations on the effect of applying to furuncles requiring evacuation a plaster consisting of soap and sugar which is used in folk-medicine for " drawing " such boils. It was found that soap and sugar applied to open boils did, in point of fact, induce a copious welling up of lymph from the subjacent tissues. In pondering this effect it suggested itself that the sugar constituent of the plaster might be attracting, or to use the household word, " drawing," fluid from the open lymph spaces ; and that the soap constituent might be decalcifying and preventing the coagulation of the out-flowing lymph by staving off the sealing of the wound by scab.

Acting upon this idea—and, of course, mindful of the fact that Heidenhain had found that sodium chloride and other crystalline substances introduced into the blood call forth an increased lymph flow—I substituted sodium chloride for the sugar, and citrate of soda for the soap ; and proceeded to treat

* 'Roy. Soc. Proc.,' A, vol. 113, p. 478 (1926).