J. Phys. B: At. Mol. Opt. Phys. **29** (1996) 4179–4192. Printed in the UK

Rainbow and glory effects in the autoionization of atoms excited by the impact of heavy ions

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Received 4 April 1996, in final form 5 August 1996

Abstract. When the autoionizing state formed by the impact of a slow positively charged ion decays, the line profile of the emitted electron is affected by the post-collision interaction with the Coulomb field of the outgoing projectile, displaying a strong enhancement in the forward direction. On the other hand, when the autoionization process is induced by a negatively charged projectile, we demonstrate that the electron spectrum displays a depletion in the forward direction and a sharp enhancement at a certain intermediate angle. We show that these structures can be interpreted as forward glory and rainbow effects, respectively. We discuss the similarities and discrepancies between a continuum-distorted wave (CDW) approximation and a classical description of these phenomena.

1. Introduction

When an atom is excited to an autoionizing state, there is a spontaneous emission of electrons with a sharply defined energy and a characteristic angular distribution. However, if the autoionizing state is formed by the impact of a slow charged ion, the spectral line shape of the emitted electron is strongly affected by the post-collision interaction (PCI) with the Coulomb field of the outgoing projectile. This effect was first observed and explained on classical grounds by Barker and Berry (1966), who showed that for positively charged projectiles, the energy spectrum is broadened and shifted towards lower energies. A quantum model was developed by Devdariani *et al* (1977), assuming that the continuum state of the emitted electron is not affected by the projectile. The next step was given by van der Straten and Morgenstern (1986) by including a semiclassical asymptotic distortion of the electronic wavefunction due to the projectile. These analyses concluded that the presence of the charged projectile and its state of motion distort the energy distribution of the emitted electrons.

Nevertheless, the receding projectile can produce even another effect on the electronic distribution which is not predicted by the former models. When the emitted electron moves at a velocity comparable to that of the positively charged projectile, the line profile is not only broadened and shifted, but there is also a sharp enhancement in the direction of the receding ion. This effect, predicted by Dahl *et al* (1976), was first observed by Swenson *et al* (1989) in $He^+ \rightarrow He$ collisions. They explained it by modelling the classical deflection of the emitted electron due to its Coulomb interaction with the projectile. The vanishing

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0953-4075/96/184179+14$19.50

of the element of solid angle at zero degree leads to a singularity in the forward direction; an effect which has been named *Coulomb focusing* (Swenson *et al* 1989). Including the projectile in the post-collision electron state by means of a continuum distorted wave (CDW) approximation, Barrachina and Macek (1989) derived an analytical expression for the electronic emission amplitude, which shows a similar sharp dependence on the emission angle. Kuchiev and Sheinerman (1988) used an alternative approximation for the final state which leads to results which are equivalent to those of the CDW model for heavy-ion collisions.

The aim of the present work is to focus attention on the similarities and discrepancies between the aforementioned quantum-mechanical and classical descriptions of these phenomena. The CDW model represents a fully quantum-mechanical approach. However, it is well known that many problems show features that can be described in terms of classical or semiclassical concepts (Ford and Wheeler 1959). In the present case, for instance, the enhancement of the emission profile in the forward direction can be interpreted in terms of a phenomenon well known within the classical description of potential scattering, namely, the *forward glory* effect. The only difference with the present effect is that now the classical trajectory of the electron starts at a certain distance from the force centre and is characterized by the initial emission angle instead of an impact parameter. Similarly to potential scattering, the forward glory effect arises since the element of solid angle for the initial emission angle remains finite while that for the final angle vanishes in the forward direction.

These similarities between the quantal and classical descriptions are even more striking in the case that the autoionizing state is induced by the collision with a negatively charged projectile. In this paper we show that the CDW model predicts a deep depletion of electron emission in the forward direction and an enhancement at a certain characteristic emission angle. In a classical description, the final angle passes through a minimum as the emission angle increases. The intensity, being proportional to the inverse of the corresponding derivative, diverges. In this case we can again take over the language of optics and assimilate this effect to the classical *rainbow* phenomenon.

In the following two sections we review different classical and quantum-mechanical models of autoionization in ion–atom collisions. In section 4 we define an effective cross section which describes the way the projectile influences the angular behaviour of the emitted electrons. In section 5 we show that this effective cross section presents a sharp enhancement in the forward direction when the charge of the projectile is positive. This effect occurs both in the quantum-mechanical description and in its classical approximation, and can be attributed to a forward glory effect. On the other hand, when the projectile is negatively charged, we are in the presence of a rainbow effect, as is shown in section 6.

2. Quantum-mechanical theories

We begin with a review of previous quantum-mechanical models for the distribution of ejected electrons in an autoionization process induced by collision. We present them in a unified description, although the original derivations were done independently.

We consider a collision of an atomic ion of charge Z_P and velocity $\boldsymbol{v}_P$ with a neutral atom which, after being excited to an autoionizing state, decays by ejecting an electron with velocity $\boldsymbol{v}$. The emitted electron travels in the two-centre Coulomb field of the ionized target and the receding projectile. The continuum state of the electron may be decomposed in the following way

$$\Psi_{\boldsymbol{v}} = D_{\boldsymbol{v}}(\boldsymbol{r}, \boldsymbol{R})\Psi_{\boldsymbol{v}}^{-}(\boldsymbol{r}),$$

where $\Psi_{\boldsymbol{v}}^{-}(\boldsymbol{r})$ is the continuum wavefunction of the electron moving with asymptotic velocity $\boldsymbol{v}$ in the field of the residual target ion alone, and $D_{\boldsymbol{v}}(\boldsymbol{r}, \boldsymbol{R})$ incorporates the distortion due to the electron–projectile interaction. $\boldsymbol{R}$ is the relative projectile–target coordinate. As was shown by Barrachina and Macek (1989) in a first order perturbation treatment, the autoionization transition amplitude may be written (in atomic units)

$$A = -\mathrm{i}A_0 \int_0^\infty D_{\boldsymbol{v}}^{*}(\mathbf{0}, \boldsymbol{v}_P t) t^{\mathrm{i}Z_P/v_P} \exp[\mathrm{i}(v^2/2 - v_0^2/2 + \mathrm{i}\Gamma/2)t]\,\mathrm{d}t, \tag{1}$$

where $E_0 = v_0^2/2$ and $1/\Gamma$ are the resonant energy and characteristic lifetime of the autoionizing state of the target. The amplitude A_0 accounts for the excitation to the autoionizing state produced by the projectile–target collision and the natural decay which would take place if the projectile had no influence on the ejected electrons. Its angular behaviour resembles the symmetry of the autoionizing state. On the other hand, the integral expression describes the PCI between the projectile and the emitted electron. From the squared modulus of equation (1), the autonization cross section may be calculated. The various electronic distributions obtained to date lie ultimately on different approximations of the distortion factor $D_{\boldsymbol{v}}$, although the original derivations followed different lines of reasoning.

The first approach was made by Devdariani *et al* (1977), who neglected the effect of the projectile on the electronic wavefunction by making $D_{\boldsymbol{v}} = 1$. This approximation implies that the projectile does modify the energy but not the trajectory of the emitted electron. The result is

$$\begin{aligned} A_{DOS} &= -\mathrm{i}A_0 \int_0^\infty t^{\mathrm{i}Z_P/v_P} \exp(\mathrm{i}(v^2/2 - v_0^2/2 + \mathrm{i}\Gamma/2)t)\,\mathrm{d}t \\ &= A_0 \frac{\Gamma(1 + \mathrm{i}Z_P/v_P)\exp(-\pi Z_P/2v_P)}{(v^2/2 - v_0^2/2 + \mathrm{i}\Gamma/2)^{1+\mathrm{i}Z_P/v_P}}. \end{aligned} \tag{2}$$

Therefore, the autoionization double differential cross section (DDCS) reads

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}E\,\mathrm{d}\Omega}\right|_{DOS} = \frac{\sigma_0}{4\pi\Gamma} g\left(\nu, \frac{E - E_0}{\Gamma/2}\right), \tag{3}$$

where $E = v^2/2$, $\nu = Z_P/v_P$ and σ_0 is an inherent cross section which does not include the PCI. The function g depends on dimensionless parameters, and reads

$$g(\nu, y) = \frac{\nu}{\sinh(\pi\nu)} \frac{2}{1 + y^2} \exp(-2\nu \arctan y). \tag{4}$$

As shown in figure 1, this function has a maximum at $y = -\nu$ and verifies

$$g(\nu, y) = g(-\nu, -y).$$

For $\nu \to 0$ the function g approaches a Lorentzian shape

$$g(\nu, y) \approx \frac{2/\pi}{1 + y^2},$$

which means that if the projectile exherts no influence upon the emitted electron, the velocity spectrum approaches the natural linewidth distribution. In the opposite limit $|\nu| \gg 1$, and since the maximum of $g(\nu, y)$ occurs at $y = -\nu$, we take $|y| \gg 1$, i.e. $|E - E_0| \gg \Gamma/2$. We therefore approximate $1 + y^2 \approx y^2$ and $\arctan y \approx \mathrm{sg}(y)\pi/2 - 1/y$, where we have defined the sign function $\mathrm{sg}(y) = 1$ for $y \geqslant 0$ and -1 for $y < 0$. Thus

$$g(\nu, y) \approx \frac{1}{2\nu} \left(\frac{2\nu}{y}\right)^2 \mathrm{e}^{2\nu/y} \frac{\exp(-\pi\nu\,\mathrm{sg}(y))}{\sinh(\pi\nu)},$$

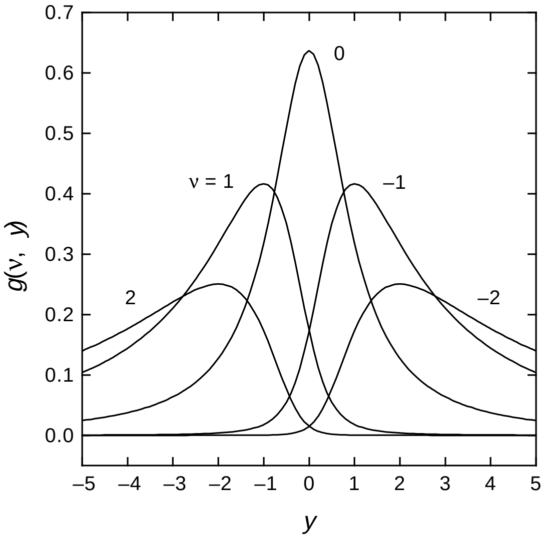

Figure 1. Electron emission intensity in the autoionization of atoms excited by the impact of heavy ions. The expression of Devdariani *et al* (1977), equation (3), is shown for different values of the dimensionless parameter $\nu = Z_P/v_P$.

which, for $|\nu| \gg 1$, reduces to

$$g(\nu, y) \approx \frac{1}{|\nu|} f(y/2\nu),$$

with

$$f(x) = \frac{1}{x^2} \mathrm{e}^{1/x} \Theta(-x),$$

where $\Theta(t)$ is the Heaviside step function. In this way, in the classical limit, the emission cross section, equation (3), reduces to

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}E\,\mathrm{d}\Omega}\right|_{BB} = \frac{\sigma_0}{4\pi\Gamma} \frac{1}{|\nu|} f\left(\frac{E - E_0}{\nu\Gamma}\right), \tag{5}$$

which is the expression obtained by Barker and Berry (1966) on phenomenological considerations. The function $f(x)$ has a maximum at $x = -\frac{1}{2}$ and a full width at half maximum of $\Delta x \approx 1.07$, describing a peak of the autoionization cross section with its maximum at an energy $E = E_0/2 - Z_P\Gamma/2v_P$ and a pronounced tail towards lower or higher energies, depending on the sign of the projectile's charge.

We therefore see that the model of Devdariani *et al* (1977), which represents the most crude approximation of the distortion factor ($D_v = 1$) leads to an electronic distribution that is shifted from $v = v_0$ and has the correct behaviour in the limits $\nu \to 0$ and $|\nu| \to +\infty$. However, this model is not consistent with the fact that, due to its infinite range, the electron–projectile Coulomb interaction does not vanish at large distances. Therefore, in the lowest-order approximation a logarithmic-phase distortion must be kept, leading to an eikonal (EI) description of the electron–projectile distortion factor

$$D_{\boldsymbol{v}}(\boldsymbol{r}, \boldsymbol{R}) \approx (v'|\boldsymbol{r} - \boldsymbol{R}| + \boldsymbol{v}' \cdot (\boldsymbol{r} - \boldsymbol{R}))^{\mathrm{i}\nu'},$$

with $\boldsymbol{v}' = \boldsymbol{v} - \boldsymbol{v}_P$ the relative electron–projectile velocity, and $\nu' = Z_P/v'$. Substituting into equation (1), we obtain

$$A_{EA} = -\mathrm{i}A_0 \int_0^\infty [(v'v_P - \boldsymbol{v}' \cdot \boldsymbol{v}_P)t]^{-\mathrm{i}\nu'} t^{\mathrm{i}\nu} \exp[\mathrm{i}(v^2/2 - v_0^2/2 + \mathrm{i}\Gamma/2)t]\,\mathrm{d}t$$
$$= \frac{A_0}{(v'v_P - \boldsymbol{v}' \cdot \boldsymbol{v}_P)^{\mathrm{i}\nu'}} \frac{\Gamma[1 + \mathrm{i}(\nu - \nu')]\exp[-\pi(\nu - \nu')/2]}{(v^2/2 - v_0^2/2 + \mathrm{i}\Gamma/2)^{-1-\mathrm{i}(\nu-\nu')}}. \tag{6}$$

Therefore, the autoionization DDCS reads

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}E\,\mathrm{d}\Omega}\right|_{EA} = \frac{\sigma_0}{4\pi\Gamma} g\left(\nu - \nu', \frac{E - E_0}{\Gamma/2}\right), \tag{7}$$

where the function $g(\nu, y)$ is given by equation (4). This electronic distribution is the same as the one obtained by van der Straten and Morgenstern (1986) and differs from that of Devdariani *et al* (1977) only by a modified projectile charge

$$Z_P^* = Z_P\left(1 - \frac{v_P}{v'}\right).$$

The discrepancies between both theories are negligible when v' is much larger than v_P. However, when $|\boldsymbol{v} - \boldsymbol{v}_P| < v_P$ the modified charge Z_P^* changes its sign. In the case of a positively charged projectile, this leads to a high energy tail in the energy spectrum of the emitted electron, instead of the usual shift to lower energies.

Except eventually for the factor σ_0, the transition rate (7) depends on the emission angle only through the modified projectile charge Z_P^*. Nevertheless, as was first observed by Swenson *et al* (1989) in $\mathrm{He}^+ + \mathrm{He}$ collisions, the interaction with the projectile can also alter the angular distribution of the emitted electrons. In order to predict this, the distortion factor D_v must give a better description of the electron–projectile interaction. This was explicitly incorporated by Barrachina and Macek (1989) by means of a CDW approximation (Cheshire 1964, Belkic *et al* 1979) of the final state of the electron. They wrote

$$D_v(\boldsymbol{r}, \boldsymbol{R}) = \Gamma(1 + \mathrm{i}\nu')\exp(\pi\nu'/2)\,{}_1F_1[-\mathrm{i}\nu', 1; -\mathrm{i}(v'|\boldsymbol{r} - \boldsymbol{R}| + \boldsymbol{v}' \cdot (\boldsymbol{r} - \boldsymbol{R})).$$

With this approximation, the transition amplitude in equation (1) can be analytically evaluated

$$A_{CDW} = -\mathrm{i}A_0 \int_0^\infty D_v^*(\mathbf{0}, \boldsymbol{v}_P t)t^{\mathrm{i}\nu} \exp(\mathrm{i}(v^2/2 - v_0^2/2 + \mathrm{i}\Gamma/2)t)\,\mathrm{d}t$$
$$= A_{DOS}\Gamma(1 - \mathrm{i}\nu')\mathrm{e}^{\pi\nu'/2}\,{}_2F_1\left(\mathrm{i}\nu'; 1 + \mathrm{i}\nu; 1; -\frac{v'v_P - \boldsymbol{v}' \cdot \boldsymbol{v}_P}{v^2/2 - v_0^2/2 + \mathrm{i}\Gamma/2}\right) \tag{8}$$

This result includes the amplitude A_{DOS}, as given by equation (2), and incorporates a dependence on the emission angle that is missed in previous theories. In particular this amplitude reduces to that of Devdariani *et al* (1977) in certain regions of the electronic velocity space (Barrachina and Macek 1989). Namely, for $v'v_P - \boldsymbol{v}' \cdot \boldsymbol{v}_P \ll \Gamma/2$ we obtain

$$A_{CDW} \approx \Gamma(1 - \mathrm{i}\nu')\mathrm{e}^{\pi\nu'/2} A_{DOS}$$

which differs from the amplitude A_{DOS} calculated by Devdariani *et al* (1977) in the normalization factor of the electron–projectile continuum state. In the limit $\boldsymbol{v}' \gg Z_P$ this Coulomb factor approaches unity and the CDW amplitude reduces to A_{DOS}. On the other hand, for $\boldsymbol{v}'\boldsymbol{v}_P - \boldsymbol{v}' \cdot \boldsymbol{v}_P \gg \Gamma/2$ we recover the semiclassical amplitude as obtained by van der Straten and Morgenstern (1986).

From equation (8), the autoionization cross section is obtained, showing a dependence on the emission angle. In particular, for a positively charged projectile, $Z_P > 0$, it predicts

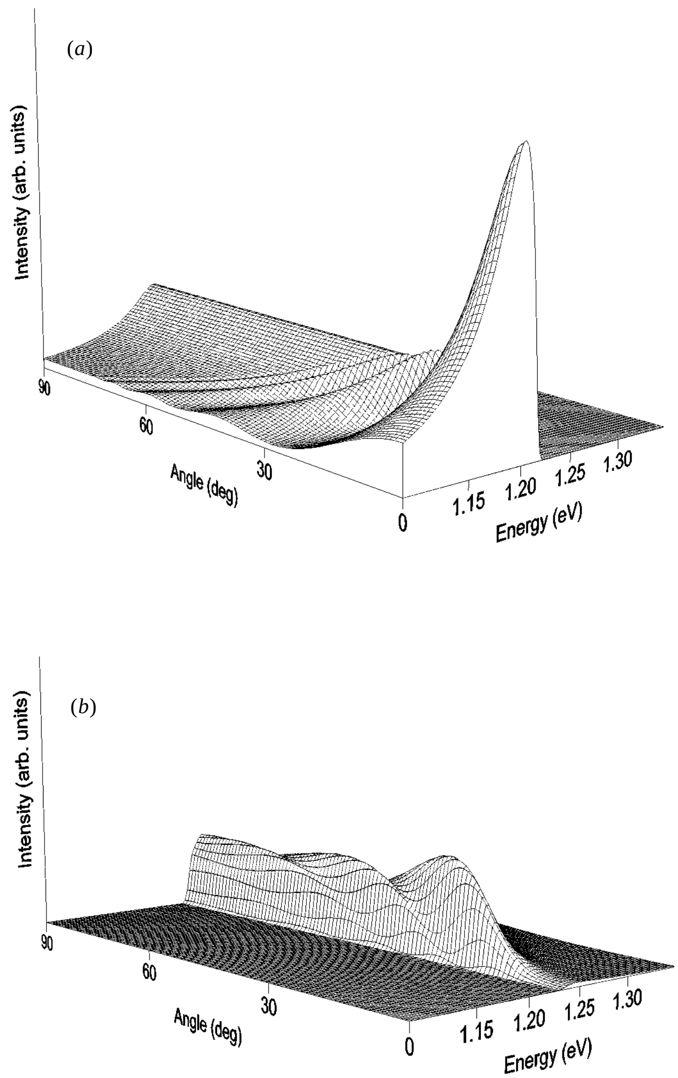

Figure 2. CDW approximation for the electron emission intensity in the autoionization of helium excited to a $(2s^2)\,^1S$ state by the impact of (*a*) a proton or (*b*) an antiproton of velocity $v_P = 0.1$ au.

a sharp enhancement in the forward direction, as shown in figure 2(*a*). This behaviour of the intensity profile was first observed by Swenson *et al* (1989) in $He^+ \to He$ collisions, who named it *Coulomb focusing*. In the next section we review a classical explanation of this effect. Furthermore, we show that the enhancement of the emission profile in the forward direction can be interpreted in terms of a phenomenon well known within the classical description of potential scattering, the *forward glory* effect. On the other hand, for a negatively charged projectile the autoionization cross section shows an enhancement at a certain characteristic angle and drops sharply in the forward direction, as shown in figure 2(*b*). The classical explanation for this effect is *rainbow* scattering, as we show in section 6.

3. Classical trajectory model

As it was pointed out by Dahl *et al* (1976), one important aspect of the autoionization process which is missing from Barker and Berry's phenomenological model refers to the deflection of the ejected electron in the Coulomb field of the projectile. This means that the presence of the charge Z_P induces a non-trivial transformation between the electronic velocity $\boldsymbol{v}_0$ at the moment of emission and the velocity $\boldsymbol{v}$ after being scattered by the projectile. In this case, some kind of dependence on the emission angle is likely to be expected. This post-collision distortion of the electron's trajectory was explicitly considered by van der Straten and Morgenstern (1986). However, a small deflection approximation prevented them from obtaining any significant dependence of the autoionization amplitude on the emission angle. Finally Swenson *et al* (1989) developed a classical generalization of Barker and Berry's model which incorporates this angle dependence by modelling the deflection of the ejected electron in the field of a positively charged projectile. In a first-order approximation, their model assumes that the electron moves in the Coulomb potential of the projectile alone. Therefore, the angle dependence of the final velocity distribution is shown to be dominated by the Jacobian $\partial(\boldsymbol{v}_0')/\partial(\boldsymbol{v}')$ of the transformation $\boldsymbol{v}_0' \to \boldsymbol{v}'$ between the initial and final velocities measured in the reference frame of the projectile. Using polar coordinates (see figure 3) and applying the energy conservation $v'^2/2 = v_0'^2/2 - Z_P/R$, the Jacobian reads

$$\frac{\partial(\boldsymbol{v}_0')}{\partial(\boldsymbol{v}')} = \frac{v_0'}{v'}\frac{\sin\alpha}{\sin\theta}\left|\frac{\mathrm{d}\alpha}{\mathrm{d}\theta}\right|.$$

The factor $\sin\alpha/\sin\theta|\mathrm{d}\alpha/\mathrm{d}\theta|$ describes the deflection of the emitted electron in the Coulomb field of the projectile. As we shall see, this term gives rise to a strong angular dependence of the emission probability which is missed in Barker and Berry's model. Actually, for a positively charged projectile and whenever $v > v_P$, this term leads to the appearence of a singularity of $\mathrm{d}\sigma/\mathrm{d}v$ in the forward direction.

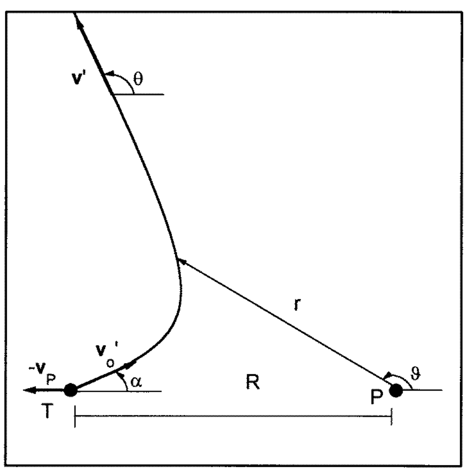

Figure 3. In the projectile's reference frame, an electron is ejected from the target T with velocity $\boldsymbol{v}_0'$ in a direction defined by the angle α, when the projectile P is at a distance R from the target. The polar coordinates of the electron are r and ϑ. Its asymptotic velocity $\boldsymbol{v}'$ makes an angle θ with the polar axis.

4. Classical effective cross section

In the previous section we have seen that the quantity

$$\Sigma_{CL}(\cos\theta) = \frac{\sin\alpha}{\sin\theta}\left|\frac{\mathrm{d}\alpha}{\mathrm{d}\theta}\right|$$

with α and θ the projectile–frame emission angles before and after scattering, respectively, describes the way the projectile influences the angular behaviour of the emitted electrons. We shall call it the *effective cross section*, and the subscript *CL* stands for *classical*. This definition is operationally equivalent to the standard one for the scattering of a beam of particles by a force centre (Landau and Lifshitz 1976)

$$\sigma(\cos\theta) = \sum_i \frac{\rho_i}{\sin\theta}\left|\frac{\mathrm{d}\rho_i}{\mathrm{d}\theta}\right|, \tag{9}$$

except that here the trajectories are parametrized not by an impact parameter ρ but by the initial emission angle α which makes this Σ_{CL} a dimensionless quantity.

In order to evaluate this effective cross section we need the transformation $\boldsymbol{v}'_0 \to \boldsymbol{v}'$. Let us consider an electron which is released with initial velocity $\boldsymbol{v}'_0$ from a target T which, at the time of emission, is located at a distance R from the projectile P. In the projectile's reference frame, the initial velocity $\boldsymbol{v}'_0$ makes an angle α with an axis through the target and the projectile. Let r and ϑ be the plane polar coordinates of the electron measured from the projectile P, as shown in figure 3. Neglecting its interaction with the residual target, the electron moves in the Coulomb potential of the projectile alone (Swenson *et al* 1989). The trajectory of the electron is given by (Laporte 1970, Samengo and Barrachina 1994)

$$\frac{R}{r}\sin^2\alpha = -\frac{A}{2}(1+\cos\vartheta) + \sin\alpha\sin(\vartheta-\alpha) \tag{10}$$

where the dimensionless parameter

$$A = \frac{-Z_P/R}{mv_0'^2/2} = \frac{v'^2}{v_0'^2} - 1$$

represents the ratio between the potential and kinetic energies at the moment of emission. We see that each particular orbit is fully characterized by this parameter A and the initial emission angle α. In particular, the eccentricity reads

$$\varepsilon = \sqrt{1 + \frac{4(A+1)}{A^2}\sin^2\alpha}.$$

In the limit $r \to +\infty$ equation (10) provides the final deflection angle $\theta = \lim_{r\to\infty}\vartheta$ in terms of the emission angle α. It reads

$$\cos\theta = -1 + \frac{\sin^2\alpha}{1 + \frac{1}{2}A - \sqrt{1+A}\cos\alpha}. \tag{11}$$

As shown in figure 4, this relation between θ and α is not one to one. There are two different initial emission angles $\alpha_\pm$ which define trajectories with the same deflection angle θ. This means that when calculating the emission rate, we have to sum up upon both branches, as given by inverting the previous equation

$$2\cos\alpha_\pm = \sqrt{1+A}(1+\cos\theta) \pm \sqrt{(1+A)(1-\cos\theta)^2 - 2A(1-\cos\theta)}. \tag{12}$$

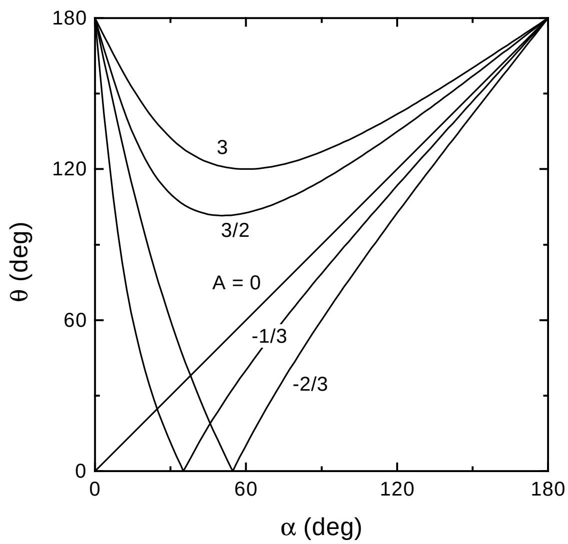

Figure 4. The final angle θ as a function of the initial angle α, as given by equation (11), for different values of the parameter $A = -2Z_P/mv_0'^2R$.

By differentiating these equations with respect to $\cos\theta$, the following expressions for the effective cross sections are obtained

$$\Sigma^{\pm}_{CL}(\cos\theta) = \left|\frac{\mathrm{d}\cos\alpha_{\pm}}{\mathrm{d}\cos\theta}\right| = \frac{1}{2}\left[\frac{1-(1+A)\cos\theta}{(2(1-\cos\theta)-(1+A)\sin^2\theta)^{1/2}} \mp \sqrt{1+A}\right]. \qquad (13)$$

In figure 5 we show the relation $\alpha(\theta)$ between the emission and deflection angles and the corresponding effective cross sections for (a) a positively charged projectile with $A = -\frac{2}{3}$ and (b) a negative one with $A = \frac{2}{3}$. In both cases, we see that $\Sigma^{\pm}_{CL}$ presents a divergency. For $Z_P > 0$ it is at $\theta = 0$ and corresponds to the so-called *forward glory effect* (Ford and Wheeler 1959). When $Z_P < 0$, we are in the presence of a *rainbow effect*, and the divergency occurs at a characteristic angle θ_R, which defines the aperture of a region in space into which none of the hyperbolic orbits penetrates. We will now analyse these phenomena separately.

5. Forward glory effect

The so called *forward glory effect* occurs in classical scattering processes whenever the dispersion function $\theta(\rho)$ passes smoothly through 0 as a function of the impact parameter ρ (Ford and Wheeler 1959). The cross section, equation (9), diverges since $\sin\theta$ vanishes while ρ and $\mathrm{d}\rho/\mathrm{d}\theta$ remain finite. In our case, we see in figure 5 and equation (13) that a similar divergence of Σ_{CL} occurs at $\theta = 0$. The explanation of this divergence is the same, except for the fact that the role of the impact parameter ρ is played by the initial emission angle α. In the scattering of a beam of particles by a force centre this effect, if present, is usually masked by the contribution from large impact parameters. This masking effect is

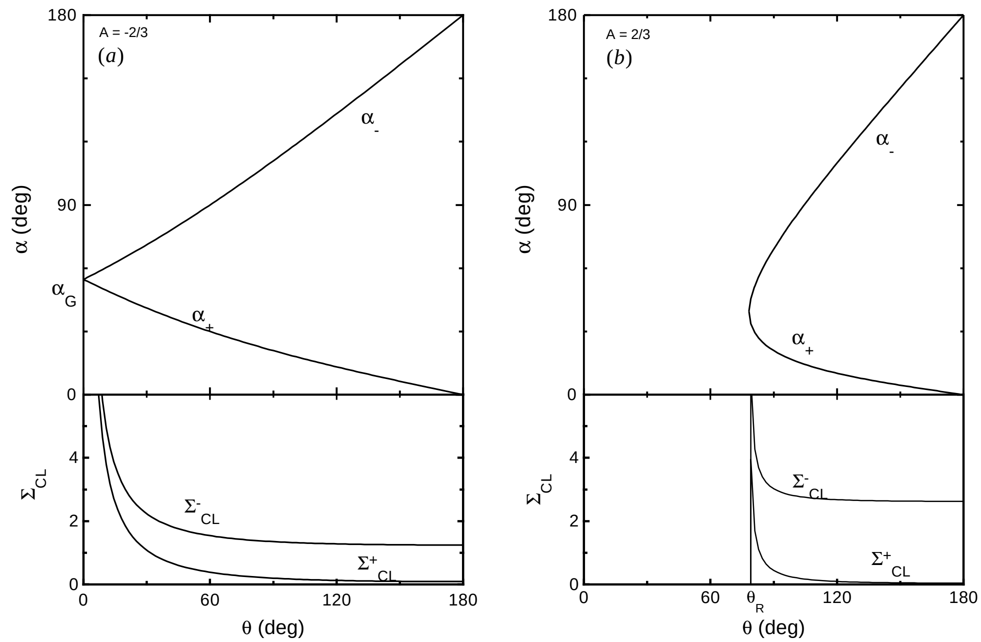

Figure 5. Dispersion relation between the emission angle α and the final angle θ, and the corresponding effective cross sections for (a) a positively charged projectile $A = -\frac{2}{3}$ and (b) a negative one $A = \frac{2}{3}$. (Note that, in order to associate the rainbow divergence with the derivative in the dispersion function, the axes α and θ have been interchanged with respect to the previous figure.) There are two emission angles $\alpha_{\pm}$ for each θ. For $Z_P > 0$ there is a forward glory divergency, and for $Z_P < 0$ the effective cross section diverges at θ_R due to a rainbow effect.

obviously absent in our case, since the glory trajectory is the only one which ends up in the forward direction.

From equation (12) we see that the deflection angle $\theta = 0$ corresponds to a so-called *glory trajectory* which is characterized by an initial emission angle $\alpha_G = \arccos\sqrt{1+A}$ shown in figure 5(a). For small values of θ, the cross section behaves as

$$\Sigma_{CL}^{\pm} \approx \frac{\sin\alpha_G}{2|\theta|}. \tag{14}$$

Physically, this divergency occurs because the attractive potential deflects those trajectories with initial emission angles near α_G into the forward direction, where the solid angle vanishes. Therefore, the angular density of trajectories is largely increased. This effect has been named *Coulomb focusing* (Swenson *et al* 1989). However, it is clear from the previous analysis that the phenomenon is not exclusive of a Coulomb interaction, and can equally happen with practically any other attractive potential. Furthermore, those electrons emitted with initial emission angles smaller than α_G are not focused in the forward direction, but end up in directions which encompass the full $(0, \pi)$ range. Hence, *glory* scattering seems to be a much more adequate name for describing this effect.

Let us now investigate this effect from a quantum mechanical point of view. From equation (8) we obtain the following expression for the autoionization DDCS

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}E\,\mathrm{d}\Omega}\right|_{CDW} = \frac{2\pi\nu'}{1-\exp\pi\nu'}\left|{}_2F_1\left(\mathrm{i}\nu'; 1+\mathrm{i}\nu; 1; -\frac{v'v_P - \boldsymbol{v}'\cdot\boldsymbol{v}_P}{E-E_0+\mathrm{i}\Gamma/2}\right)\right|^2 \left.\frac{\mathrm{d}\sigma}{\mathrm{d}E\,\mathrm{d}\Omega}\right|_{DOS}$$

where $d\sigma/dE\, d\Omega|_{DOS}$ is the autoionization cross section equation (3) obtained by Devdariani *et al* (1977). The term accompanying $d\sigma/dE\, d\Omega|_{DOS}$ accounts for the effect of the projectile in the angular behaviour of the electrons. Its dependence on the parameters of the problem is much more intricate than in the classical approach. In figure 2(a) we show its angular behaviour for an attractive interaction between the electron and the projectile, i.e. $Z_P > 0$. We see that, while Σ_{CL} in figure 5 is monotonic and presents a glory divergency in the forward direction, the quantum-mechanical DDCS oscillates, and reaches a finite value at zero degrees. Nevertheless, the global behaviour of both cross sections is similar. From equation (5) it can be seen that the forward intensity is given by the normalization factor of the Coulomb continuum wavefunction.

$$N(\nu') = 2\pi\nu'/(1 - \exp(-2\pi\nu')). \tag{15}$$

We see that this enhancement is more pronounced the greater the charge and the slower the relative electron–projectile velocity. In the classical case, the forward divergency as given by equation (14) shows a similar dependence.

It is clear that a similar divergence, called *backward glory*, could eventually occur in the backward direction, however it is not observed. This fact seems to be at odds with what happens in the scattering of a beam of particles by a force centre, where this backward glory divergence is much more easily observed than its counterpart at $\theta = 0$, since it is not masked by other spurious contributions. In our case, however, no backward glory divergence occurs since, when $\theta = \pi$, the initial solid angle $2\pi \cos\alpha$ in equation (13) vanishes together with $2\pi \cos\theta$.

It was the similarity of the backward glory phenomenon with the optical glory which prompted Ford and Wheeler (1959) to coin the term *glory scattering* (Bryant and Jarmie 1974), even though the correspondence between both mechanisms is not as clear as for the rainbow phenomenon, as we shall see in the following section.

6. Rainbow effect

As a corollary of the ideas presented up to now, it can be foreseen that an interesting effect is likely to occur if the autoionizing process is induced by a negatively charged projectile. As a result of a rainbow phenomenon, a deep depletion of electron emission in the forward direction and a sharp enhancement at a certain characteristic emission angle θ_R have to be expected. We see in figure 2(b) that this is actually the case. Once again, there are some similitudes and some differences between the classical and the quantum mechanical descriptions. For instance, the rainbow divergency of the classical picture becomes a maximum in the quantum-mechanical approach. Furthermore, the angle of this maximum is slightly greater than θ_R. For angles smaller than θ_R, the classical cross section is strictly zero, while the quantum-mechanical one shows a pronounced minimum.

In atomic scattering, singularities in the classical cross section were first predicted by Firsov (1953) and independently by Manson (1957). By analogy with the corresponding optical effect, this phenomenon was called *rainbow scattering* by Ford and Wheeler (1959). Since these early papers and the first observation by Beck (1962), the rainbow effect has been widely observed in atomic collisions (Kleyn 1987). These atomic rainbows are again due to the fact that, whenever the dispersion relation $\theta(\rho)$ shows an extremum, there is a singularity in the scattering cross section, equation (9), of the form

$$\sigma(\cos\theta) \propto \frac{1}{\sqrt{|\theta - \theta_R|}}.$$

In our case, the divergence of Σ_{CL} for the case of a negatively charged projectile occurs at an angle

$$\theta_R = 2\arctan(\sqrt{A}). \qquad (16)$$

The cause of this divergence lies in the fact that, as the emission angle α is varied from 0 to π, the deflection angle θ diminishes from π to a minimum value θ_R, and thereafter increases again. The effective cross section $\Sigma_{CL}(\cos\theta)$, being proportional to $(\mathrm{d}\theta/\mathrm{d}\alpha)^{-1}$, becomes infinite at precisely this minimum angle θ_R. For θ close to θ_R the classical effective cross section behaves like

$$\Sigma_{CL}^{\pm} \approx \frac{\sqrt{\tan(\theta_R/2)}}{4\cos(\theta_R/2)} \frac{1}{\sqrt{\theta - \theta_R}} \Theta(\theta - \theta_R).$$

This phenomenon is geometrically analogous to what happens to a beam of light rays scattered by a water droplet, leading to the formation of an optical rainbow. Hence, this kind of divergency is known as *rainbow scattering* (Ford and Wheeler 1959). Usually, rainbows are due to a balance of counteracting forces. This means that they are met in the case of potentials which are repulsive in one part of space and attractive in some other part. This condition is necessary if the scattering is such that the particles come from infinity. In our case, the same effect is achieved by only one force. The difference is that, instead of having an initial distribution with the geometry of a parallel beam, we have an outgoing flux diverging from one point.

The rainbow in the sky was one of the first physical phenomena to be studied scientifically. The first explanation of the rainbow in the modern conception of science was given by René Descartes in 1637 (see the review paper by Nussenzveig (1977)). He discovered that when a beam of light enters a water droplet and suffers one internal reflection there is an extremum angle θ_R where the density of trajectories diverges, producing a clear bow in the sky. The colours of the rainbow were explained thirty years later by Newton in his prism experiment. Each colour has a different index of refraction by water and, therefore, a slightly different rainbow angle. This picture gives rise to a divergency of the light intensity at the rainbow angle and a dark zone (*Alexander's dark band*) for smaller angles. Taking into account the possibility of more than one internal reflection, higher-order rainbows can be predicted. As a matter of fact, Alexander's dark band is the dark stripe between the first- and second-order rainbows. In figure 2(*b*) we see that a similar *dark band* is also observed in our effective cross section for $\theta < \theta_R$. Actually the Heaviside step function in equation (13) vanishes for $\theta < \theta_R$, making the effective cross section zero in this *excluded region*.

When the interaction between the projectile and the emitted electron increases, the shadow zone becomes broader both in the classical and quantum-mechanical descriptions. Regarding the forward intensity of the quantum mechanical cross section, we can still use equation (15). As the exponential has now a positive argument, the forward intensity will be smaller the greater is $|Z_P|$ and the smaller is v'. In any case, it is clear that the number of particles emitted in the forward direction is no longer zero, as in the classical case.

This non-zero intensity inside Alexander's dark band represents an important difference between the quantum mechanical treatment and its classical approximation. Similarly, the classical picture of the rainbow phenomenon, as given by Descartes and Newton, suffers from this same inconsistency. A partial answer was given by Airy in 1838, when he calculated the evolution of a wavefront inside a water droplet. Instead of a rainbow divergency, he obtained a maximum located at a slightly greater angle with a continuous transition to the dark band. The fact that Alexander's dark band was no longer strictly dark was associated to a diffraction effect into the geometrical shadow. In our system, the situation is completely analogous.

Another feature which could not be explained with the classical picture was the interference phenomenon. Sometimes, several luminous arcs can be observed under the principal rainbow. These arcs are called *supernumerary arcs*. Their explanation had to wait until 1803, when Thomas Young gave an undulatory description of the rainbow. As shown in figure 4, there are two distinct paths which end up in any given direction which can give rise to constructive or destructive interference, depending on the observation angle. This accounts for the intensity oscillations of the supernumerary arcs. In the same way, the CDW description of the autoionization process incorporates oscillations which were not present in the classical description. This interference phenomenon is also described by a semiclassical model developed by Swenson *et al* (1991). The similarity between both descriptions relies on the semiclassical concept of nearside and farside scattering (Samengo and Barrachina 1996), which was first used to describe elastic reactions of spin-zero nuclei (Fuller 1975).

7. Conclusions

We have considered an autoionization process induced by the collision of a charged heavy ion with a neutral atom. We studied the post-collisional effects produced by the interaction between the ejected electron and the outgoing projectile by defining an effective cross section which gives the angular distribution of the emitted electrons. In a classical description, this effective cross section diverges in the forward direction for a positively charged projectile. This structure can be interpreted in terms of a forward glory mechanism. The same effect is also observed in a quantum-mechanical CDW approach and with a similar global behaviour, except for the fact that the divergency turns into a maximum and the cross section exhibits angular oscillations which are not present in the classical picture. These similarities and differences are not exclusive of this particular system, but are present whenever corpuscular and undulatory formulations of a given phenomenon are compared.

On the other hand, we showed that if the charge of the projectile is negative, the autoionization cross section exhibits a very interesting effect characterized by a deep depletion of the forward electron emission and an enhancement at a certain intermediate angle. By comparison with a classical description we were able to ascribe these effects to a rainbow phenomenon. The explanation of both characteristics is similar to that of the rainbow arc and Alexander's dark band in the optical effect.

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