The role of classical uncertainties on the coherence properties of collisions processes

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Synopsis: The awareness of the relevance of the projectile's beam coherence effects in ion-atom and ion-molecule collisions has motivated an extensive study on this subject in recent years. The traditional way to compute the fully differential cross section (FDCS) of a reaction describes the incoming beam of projectiles as a purely coherent quantum system, i.e. a plane-wave. Nevertheless, this assumption seems not justified enough when we look at this postulate under the light of the density matrix theory. We describe how the time propagation of an incoherent mixture of an ensemble of massive particles, described by identical wave packets located at different positions upon a much larger region, can eventually develop coherence at a given distance L, and its implications for scattering theory. Furthermore, we show the results of these concepts applied to specific experimental results.

INTRODUCCIÓN

No matter how much we struggle to reduce the classical uncertainties in a collision experiment, we could never suppose them null. In other words, it would never be possible to describe all the different scattering events by a single and unique wave function. However, this is what we do in the standard scattering theory, and even assume that this wave function can be represented by a plane wave [1].

The incompleteness of describing any wave packet as a plane wave was first realized in the context of Optics, and the region of validity of this approximation was established. Direct sunlight exhibits spatial coherence over a length of tens of μm at the Earth surface [1], even though it is produced by a conspicuously incoherent source, the Sun. This effect, where the light emitted from an incoherent source becomes approximately coherent at large distances, was demonstrated by Pieter H. van Cittert [2] in 1934 (see also [3]).

The study of the equivalent coherence properties of particle beams has been recently validated by means of a quantum mechanical approach and applied in the context of scattering theory.

The key element for unraveling this conundrum is the coherence length,

$$\ell_{\Delta p\,\Delta b} = \hbar\sqrt{\frac{1}{(\Delta k)^2+(\Delta p)^2}+\frac{(L/p)^2}{(\Delta x)^2+(\Delta b)^2}}$$

where $(\Delta x; \Delta k)$ and $(\Delta b; \Delta p)$ are the quantum and classical uncertainties in position and momentum respectively, p is the initial relative projectile-target momentum, and L is the distance freely traveled by the projectile before reaching the target.

Now, the standard "fully coherent" scattering theory applies whenever $\ell_{\Delta p\,\Delta b}$ is much larger than any characteristic size of the scattering event [2, 3, 4]. Otherwise, an incoherent (or, at least, partially coherent) calculation is mandatory.

In recent years, a series of experiments have explored this transition between coherent and incoherent collisions (see, eg. [5]), by modifying the different parameters in the previous equation. Some of them have successfully achieved that by modifying the length L [6], i.e by controlling the angle θ_L= arctan($\Delta b/L$) subtended by the collimator. Other authors [7, 8] have tried to set the focusing of the projectile's beam, as characterized by its angular dispersion θ_p =arctan($\Delta p/p$).

RESULTS

In order to understand the role of classical uncertainties in modifying the coherence length, we will rewrite Eq. (1) as

$$\frac{\ell_{\Delta p\,\Delta b}}{\hbar/p} = \sqrt{\frac{1}{\tan^2(\theta_{\Delta p}/2)}+\frac{1}{\tan^2(\theta_{\Delta b}/2)}} \quad \text{with} \quad \frac{\theta_{\Delta p}}{2} = \arctan\frac{\sqrt{(\Delta k)^2+(\Delta p)^2}}{p} \quad \text{and} \quad \frac{\theta_{\Delta b}}{2} = \arctan\frac{\sqrt{(\Delta x)^2+(\Delta b)^2}}{L}$$

Here $\theta_{\Delta p}$ defines the focusing of the beam. For a perfectly focused beam (i.e. for $\Delta p = 0$), we obtain $\theta_{\Delta p}$= 2Δk =p, while in the opposite limit, $\theta_{\Delta p} = \pi$. On the other hand, $\theta_{\Delta b}$ may vary between 0 for L→ ∞ (i.e. when the source is at an infinite distance from the target), and π for Δb = 1 (i.e. a plane wave).

Note in Figure 1 that, since the focusing angle $\theta_{\Delta p}$ is limited by $\theta_{\Delta p} \geq 2\Delta k = p$, it can not -by itself- increase the coherence length up to macroscopic values; while $\theta_{\Delta b}$, being unlimited due to the van Cittert-Zernike theorem, can. In Figure 2 we show a different representation of this effect where we assume (in atomic units) Δx = 1 and p = 1.

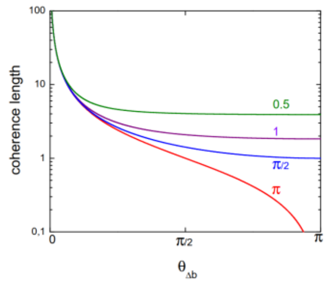

Figure 1. Coherence length $\ell_{\Delta p\,\Delta b}$ as a function of $\theta_{\Delta b}$ for various values of $\theta_{\Delta p}$

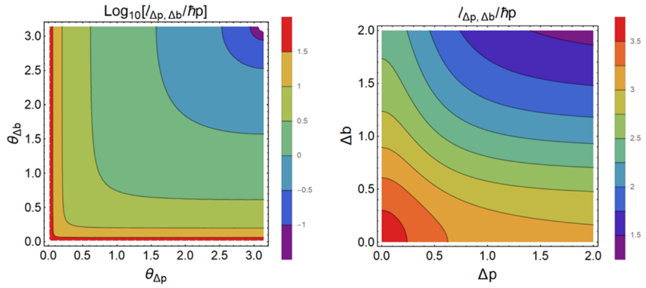

Figure 2. Coherence length $\ell_{\Delta p\,\Delta b}$ as a function of and $\theta_{\Delta b}$ and $\theta_{\Delta p}$ (left), and as a function of Δb and Δp (right).

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CONCLUSIONS

We have discussed the effects that the uncertainties (of classical origin) in the position and the initial impulse of the quantum states might produce on the coherence of an ensemble of particles. In this sense, this work is complementary to a previous article [4], dedicated to the study of the van Cittert-Zernike effect for the case where only a classical distribution of the initial position was considered, without considering possible uncertainties in the impulse.